

Making Neutrinos Massive with an Axion in Supersymmetry

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Abstract

The minimal supersymmetric standard model (MSSM) of particle interactions is extended to include three singlet (right-handed) neutrino superfields together with three other singlet superfields. The resulting theory is assumed to be invariant under an anomalous global U(1) (Peccei-Quinn) symmetry with one fundamental mass m_2 . The soft breaking of supersymmetry at the TeV scale is shown to generate an axion scale f_a of order m_2 . Neutrino masses are generated by f_a according to the usual seesaw mechanism.

The minimal supersymmetric standard model (MSSM) is a well-motivated extension of the standard model (SM) of particle interactions. Nevertheless, it is missing at least two important ingredients. There is no neutrino mass and the strong CP problem [1] is unresolved. Whereas separate remedies exist for both shortcomings, they are in general unrelated [2]. In the following, I start with a supersymmetric theory of just one large fundamental mass (m_2). I assume it to be invariant under an anomalous global U(1) symmetry which is an extension of the well-known Peccei-Quinn symmetry [3] to include three singlet (right-handed) neutrino superfields (\hat{N}^c) and three other singlet superfields (\hat{S}). The supersymmetry is then softly broken at M_{SUSY} of order 1 TeV. As a result of the assumed particle content of the theory, an axion scale f_a of order m_2 is generated, from which neutrinos obtain masses via the usual seesaw mechanism with $m_N \sim f_a$.

Consider first the MSSM superpotential:

$$\hat{W} = \mu \hat{H}_u \hat{H}_d + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c + h_e \hat{H}_d \hat{L} \hat{e}^c. \quad (1)$$

Under $U(1)_{PQ}$, the quark ($\hat{Q}, \hat{u}^c, \hat{d}^c$) and lepton (\hat{L}, \hat{e}^c) superfields have charges $+1/2$, whereas the Higgs (\hat{H}_u, \hat{H}_d) superfields have charges -1 . Hence the μ term is forbidden. It is replaced here by $h_2 \hat{S}_2 \hat{H}_u \hat{H}_d$, where \hat{S}_2 is a singlet superfield with PQ charge $+2$.

Add three singlet superfields \hat{N}^c with PQ charges $+1/2$. The term $h_N \hat{H}_u \hat{L} \hat{N}^c$ is then allowed, but the usual Majorana mass term $m_N \hat{N}^c \hat{N}^c$ is forbidden. Instead, it is replaced by $h_1 \hat{S}_1 \hat{N}^c \hat{N}^c$, where \hat{S}_1 has PQ charge -1 .

So far there is no mass scale in the superpotential of this theory. It is thus natural to introduce a third singlet superfield \hat{S}_0 with PQ charge -2 so that the complete superpotential of this theory is given by

$$\begin{aligned} \hat{W} = & m_2 \hat{S}_2 \hat{S}_0 + f \hat{S}_1 \hat{S}_1 \hat{S}_2 + h_1 \hat{S}_1 \hat{N}^c \hat{N}^c + h_2 \hat{S}_2 \hat{H}_u \hat{H}_d \\ & + h_N \hat{H}_u \hat{L} \hat{N}^c + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c + h_e \hat{H}_d \hat{L} \hat{e}^c. \end{aligned} \quad (2)$$

The mass m_2 is a large fundamental scale which will be shown to coincide with the axion scale, even though supersymmetry is only broken at the TeV scale. The term $\hat{S}_1\hat{S}_1\hat{S}_2$ is automatically present and will be the key to understanding how f_a is generated from M_{SUSY} . Consider next the spontaneous breaking of $U(1)_{PQ}$ by the vacuum expectation values $v_{2,1,0}$ of $S_{2,1,0}$ respectively. The μ term of the MSSM is then given by h_2v_2 and the Majorana mass of the neutrino singlet is $2h_1v_1$. Hence v_1 should be many orders of magnitude greater than v_2 . With $m_N = 2h_1v_1$, the usual seesaw relationship

$$m_\nu = \frac{h_N^2 v_u^2}{m_N} \quad (3)$$

is also obtained. Now the axion scale f_a is of order v_1 as well, thus $m_N \sim f_a$. Whereas this relationship was also proposed previously [2], the hierarchy problem of $v_2 \ll v_1$ was not addressed. If a sterile neutrino superfield $\hat{\nu}_s$ is desirable, it may be assigned PQ charge $-5/2$. The term $h_s\hat{S}_2\hat{\nu}_s\hat{N}^c$ is then allowed in Eq. (2), resulting in a seesaw mass for ν_s given by $h_s^2 v_2^2 / m_N$.

The strong CP problem is the problem of having the instanton-induced term [1]

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta} \quad (4)$$

in the effective Lagrangian of quantum chromodynamics (QCD), where g_s is the strong coupling constant, and

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g_s f_{abc} G_b^\mu G_c^\nu \quad (5)$$

is the gluonic field strength. If θ_{QCD} is of order unity, the neutron electric dipole moment is expected [4] to be 10^{10} times its present experimental upper limit ($0.63 \times 10^{-25} e \text{ cm}$) [5]. This conundrum is most elegantly resolved by invoking a dynamical mechanism [3] to relax the above θ_{QCD} parameter (including all contributions from colored fermions) to zero. However, this necessarily results [6] in a very light pseudoscalar particle called the axion, which has not yet been observed [7].

To reconcile the nonobservation of an axion in present experiments and the constraint $10^9 \text{ GeV} < f_a < 10^{12} \text{ GeV}$ from astrophysics and cosmology [8], three types of “invisible” axions have been discussed. The DFSZ solution [9] introduces a heavy singlet scalar field as the source of the axion but its mixing with the doublet scalar fields (which couple to the usual quarks) is very much suppressed. The KSVZ solution [10] also has a heavy singlet scalar field but it couples only to new heavy colored fermions. The gluino solution [11] identifies the $U(1)_R$ of superfield transformations with $U(1)_{PQ}$ and thus the axion is a dynamical phase attached to the gluino as well as all other superparticles. The present model is of the DFSZ type, but the use of 3 fundamental superfields is motivated by Refs.[12, 13].

Before discussing the spontaneous breaking of $U(1)_{PQ}$ in the context of soft supersymmetry breaking, consider \hat{W} of Eq. (2) in terms of baryon number and lepton number. It is clear that the former is conserved as a global symmetry (\hat{Q} has $B = 1/3$, \hat{u}^c and \hat{d}^c have $B = -1/3$, all others have $B = 0$), whereas the latter is conserved only as a discrete symmetry (\hat{L} , \hat{e}^c , and \hat{N}^c are odd, all others are even). Thus the usual R parity of the MSSM is also conserved. The three \hat{N}^c superfields are well-motivated because they allow small seesaw neutrino masses for neutrino oscillations [14, 15, 16]. The Peccei-Quinn symmetry is well-motivated as the most attractive solution of the strong CP problem. Hence S_1 and S_2 are both well-motivated. Finally, S_0 is well-motivated because \hat{W} should have a large fundamental mass scale. Given all these well-motivated inputs, Eq. (2) is uniquely determined and the two crucial extra terms $m_2 \hat{S}_2 \hat{S}_0$ and $f \hat{S}_1 \hat{S}_1 \hat{S}_2$ are automatically present [17].

Consider the scalar potential of $S_{2,1,0}$, i.e.

$$V = |m_2 S_0 + f S_1^2|^2 + m_2^2 |S_2|^2 + 4f^2 |S_1|^2 |S_2|^2. \quad (6)$$

There are two supersymmetric minima: the trivial one with $v_0 = v_1 = v_2 = 0$, and the much more interesting one with

$$v_2 = 0, \quad m_2 v_0 + f v_1^2 = 0, \quad (7)$$

where $v_{2,1,0} = \langle S_{2,1,0} \rangle$. The latter breaks $U(1)_{PQ}$ spontaneously and the superpotential of $\hat{S}_{2,1,0}$ becomes

$$\hat{W}' = \frac{m_2}{v_1}(v_1\hat{S}_0 - 2v_0\hat{S}_1)\hat{S}_2 + f\hat{S}_1\hat{S}_1\hat{S}_2, \quad (8)$$

after shifting by $v_{2,1,0}$. This shows clearly that the linear combination

$$\frac{v_1\hat{S}_1 + 2v_0\hat{S}_0}{\sqrt{|v_1|^2 + 4|v_0|^2}} \quad (9)$$

is a massless superfield. Hence the axion is not even contained in \hat{S}_2 , and its effective coupling through \hat{S}_2 will be suppressed by $M_{SUSY}/v_{1,0}$ as desired.

At this point, the individual values of v_1 and v_0 are not determined. This is because the vacuum is invariant not only under a phase rotation but also under a scale transformation as a result of the unbroken supersymmetry [18]. As such, it is unstable and the soft breaking of supersymmetry at the TeV scale will determine v_1 and v_0 , and v_2 will become nonzero. Specifically, the supersymmetry of this theory is assumed broken by all possible holomorphic soft terms which are invariant under $U(1)_{PQ}$. In particular, all the usual MSSM soft terms are present except for the $\mu B H_u H_d$ term. However, there is the $h_2 A_2 S_2 H_u H_d$ term as well as the $|m_2 S_0 + f S_1 S_1 + h_2 H_u H_d|^2$ term, hence “ μB ” = $h_2[A_2 v_2 + (m_2 v_0 + f v_1^2)]$. Recall that the μ parameter of the MSSM is replaced here by $h_2 v_2$. Hence v_2 should be of order M_{SUSY} and $m_2 v_0 + f v_1^2$ of order M_{SUSY}^2 , and that is exactly what will be shown in the following.

Add now the other holomorphic soft terms of the scalar potential:

$$\begin{aligned} V_{soft} &= \mu_0^2 |S_0|^2 + \mu_1^2 |S_1|^2 + \mu_2^2 |S_2|^2 \\ &+ [\mu_{20} m_2 S_2 S_0 + \mu_{12} S_1^2 S_2 + h.c.], \end{aligned} \quad (10)$$

where all new parameters are assumed of order $M_{SUSY} \sim 1$ TeV. Consider the minimum of the scalar potential of $S_{2,1,0}$ with the addition of V_{soft} , i.e.

$$V_{min} = (m_2^2 + \mu_0^2)v_0^2 + \mu_1^2 v_1^2 + (m_2^2 + \mu_2^2)v_2^2 + 2\mu_{20} m_2 v_2 v_0$$

$$+ 2m_2 f v_1^2 v_0 + 2\mu_{12} v_1^2 v_2 + 4f^2 v_2^2 v_1^2 + f^2 v_1^4, \quad (11)$$

where every quantity has been assumed real for simplicity. The conditions on $v_{2,1,0}$ are

$$(m_2^2 + \mu_2^2 + 4f^2 v_1^2) v_2 + \mu_{20} m_2 v_0 + \mu_{12} v_1^2 = 0, \quad (12)$$

$$f(m_2 v_0 + f v_1^2) + \frac{1}{2} \mu_1^2 + \mu_{12} v_2 + 2f^2 v_2^2 = 0, \quad (13)$$

$$m_2(m_2 v_0 + f v_1^2) + \mu_0^2 v_0 + \mu_{20} m_2 v_2 = 0. \quad (14)$$

Together they show that Eq. (7) is modified to read

$$v_2 \sim M_{SUSY}, \quad m_2 v_0 + f v_1^2 \sim M_{SUSY}^2. \quad (15)$$

However, v_0 and v_1 are individually of order m_2 . This can be shown by integrating out the heavy fields $S_{2,0}$ and $\tilde{S}_{2,0}$ in the limit where S_1 is massless. The effective scalar potential for S_1 is necessarily of the form

$$V_{eff} = a |S_1|^2 + b |S_1|^4. \quad (16)$$

At tree level, $a = \mu_1^2$, and

$$b = f^2 - \frac{(m_2 f)^2}{m_2^2 + \mu_0^2} - \frac{\mu_{12}^2}{m_2^2 + \mu_2^2} \simeq \frac{f^2 \mu_0^2 - \mu_{12}^2}{m_2^2}, \quad (17)$$

where the mixing parameter μ_{20} has been neglected. Hence

$$v_1^2 = \frac{-a}{2b} \simeq \frac{-\mu_1^2 m_2^2}{2(f^2 \mu_0^2 - \mu_{12}^2)}. \quad (18)$$

Since all soft supersymmetry breaking parameters are of order M_{SUSY} , this shows that v_1 is of order m_2 and so is v_0 . The parameters a and b have logarithmically divergent corrections in one loop, but they are proportional to M_{SUSY}^2 and M_{SUSY}^2/m_2^2 respectively, hence they do not spoil the tree-level result of Eq. (18). [This would not be the case if the nonholomorphic soft term $S_1^2 S_0^*$ were added.]

To understand why $v_1 \gg M_{SUSY}$ is possible, consider the superfield of Eq. (9). The phase of the corresponding scalar field is the axion, but the magnitude is a physical scalar

particle of mass $\sim M_{SUSY}$, and the associated Majorana fermion (axino) also has a mass $\sim M_{SUSY}$. Hence $v_1 \neq 0$ does not necessarily imply that supersymmetry is broken at that scale. For example, the superfield N^c has the large mass $2h_1 v_1$, and M_{SUSY} accounts only for the relative small splitting between its scalar and fermion components.

As the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry is broken by the vacuum expectation values $v_{u,d}$ of $H_{u,d}$, the observed doublet neutrinos acquire naturally small Majorana masses given by $m_\nu = h_N^2 v_u^2 / (2f v_1)$ via the usual seesaw mechanism. Since $H_{u,d}$ have PQ charges as well, the axion field is now given by

$$a = \frac{v_1 \theta_1 + 2v_0 \theta_0 - 2v_2 \theta_2 + v_u \theta_u + v_d \theta_d}{V} \quad (19)$$

where $V = \sqrt{v_1^2 + 4v_0^2 + 4v_2^2 + v_u^2 + v_d^2}$, and θ_i are the various properly normalized angular fields, from the decomposition of a complex scalar field $\phi = (1/\sqrt{2})(v + \rho) \exp(i\theta/v)$ with the kinetic energy term

$$\partial_\mu \phi^* \partial^\mu \phi = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{2} (\partial_\mu \theta)^2 \left(1 + \frac{\rho}{v}\right)^2. \quad (20)$$

The axionic coupling to quarks is thus

$$(\partial_\mu a) \left[\frac{1}{2} \left(\frac{v_u}{V} \right) \frac{1}{v_u} \bar{u} \gamma^\mu \gamma_5 u + \frac{1}{2} \left(\frac{v_d}{V} \right) \frac{1}{v_d} \bar{d} \gamma^\mu \gamma_5 d \right] = \frac{1}{2V} (\partial_\mu a) \sum_{q=u,d} \bar{q} \gamma^\mu \gamma_5 q, \quad (21)$$

as in the DFSZ model.

Consider now all the physical particles of this theory. (1) There is a heavy Dirac fermion of mass $m_2 \sqrt{1 + 4v_0^2/v_1^2}$, formed out of \tilde{S}_2 and a linear combination of \tilde{S}_0 and \tilde{S}_1 . The two associated scalars also have the same mass but with vacuum expectation values of order M_{SUSY} . (2) There are three heavy N^c superfields with mass of order $\langle S_1 \rangle = v_1 \sim m_2$. They provide seesaw masses for the neutrinos and generate a primordial lepton asymmetry through their decays [19]. This gets converted into the present observed baryon asymmetry of the Universe through the $B + L$ violating electroweak sphalerons [20]. (3) The particles

of the MSSM and their interactions are all present, but with the μ parameter given by $h_2 v_2 \sim M_{SUSY}$ and the μB parameter by $h_2[A_2 v_2 + (m_2 v_0 + f v_1^2)] \sim M_{SUSY}^2$, thus solving the μ problem (i.e. why $\mu \sim M_{SUSY}$ and not m_2) without causing a μB problem. (4) Whereas the spontaneous breaking of $U(1)_{PQ}$ generates an axion at the scale $V \sim m_2$, thus solving the strong CP problem, the physical scalar field (saxion) with this dynamical phase has a mass $\sim M_{SUSY}$. It is effectively unobservable [12] because its couplings to all MSSM particles are suppressed by at least $v_{2,u,d}/V$. (5) An axino of mass $\sim M_{SUSY}$ also exists. Since R parity is conserved and the axino has $R = -1$, it may be the stable LSP (lightest supersymmetric particle) of this theory and be experimentally observed. [21]

The 7×7 mass matrix spanning the $R = -1$ neutral fermions of this theory in the basis $(\tilde{B}, \tilde{W}_3, \tilde{H}_u^0, \tilde{H}_d^0, \tilde{S}_2, \tilde{S}_0, \tilde{S}_1)$ is given by

$$\mathcal{M} = \begin{bmatrix} \tilde{m}_1 & 0 & -sm_3 & sm_4 & 0 & 0 & 0 \\ 0 & \tilde{m}_2 & cm_3 & -cm_4 & 0 & 0 & 0 \\ -sm_3 & cm_3 & 0 & h_2 v_2 & h_2 v_d & 0 & 0 \\ sm_4 & -cm_4 & h_2 v_2 & 0 & h_2 v_u & 0 & 0 \\ 0 & 0 & h_2 v_d & h_2 v_u & 0 & m_2 & 2f v_1 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2f v_1 & 0 & 2f v_2 \end{bmatrix}, \quad (22)$$

where $s = \sin \theta_W$, $c = \cos \theta_W$, $m_3 = M_Z \cos \beta$, $m_4 = M_Z \sin \beta$, with $\tan \beta = v_u/v_d$. Without $\tilde{S}_{2,1,0}$, the above is just the neutralino mass matrix of the MSSM and the LSP is a linear combination of the two gauginos and the two Higgsinos. In this theory, that combination has a small overlap with the axino of order $v_{u,d}/V$. If kinematically allowed, it will decay into the axino and the Z boson or a neutral Higgs boson.

In conclusion, a desirable extension of the MSSM has been presented which has only two input scales, i.e. the large fundamental scale m_2 and the soft supersymmetry breaking scale M_{SUSY} . Assuming the validity of $U(1)_{PQ}$ and its implementation in terms of Eqs. (2) and (10), an axion scale $\sim m_2$ is generated, which solves the strong CP problem and makes

neutrinos massive via the usual seesaw mechanism, without breaking the supersymmetry at m_2 . The baryon asymmetry of the Universe is accommodated as well as the existence of dark matter. The μ problem of the MSSM is solved without causing a μB problem. The other particles associated with the axion (saxion and axino) have masses of order M_{SUSY} , with the axino as a candidate for the true lightest supersymmetric particle.

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